Bootstrapping the Relative Performance of Yield Curve Strategies

Razvan Pascalau and Ryan Poirier*

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Abstract

The present study assesses the relative performance of yield curve strategies involving bullet and barbell portfolios due to changes in the shape of the yield curve via shocks to the Dow Jones index. We employ three different yield curve models and bootstrap the bond portfolio performance using a block bootstrap approach to compute the 66 percent confidence intervals. We allow for co-movement among the yield curve factors. The study finds that a new parametrization we propose yields tighter confidence intervals than the usual approaches. In addition, we show that the shape of the confidence curves with respect to changes in terms to maturity, coupon rates, and market changes depends on the choice of the yield curve parametrization. This finding yields several important implications for bond portfolio strategies.

JEL: Bootstrapping, Yield Curve, Barbell and Bullet Portfolios Keywords: G11, G12

*Corresponding author - Pascalau: SUNY Plattsburgh, 101 Broad Street, Redcay 124, rpasc001@plattsburgh.edu. Poirier: NYU Polytechnic School of Engineering, rp1848@nyu.edu. We thank Gregory Quenell and an anonymous referee for excellent feedback and comments on an earlier version of the paper. Any remaining errors are our own.
Optimal bond portfolio strategies require the ability of practitioners and researchers to simulate the changes in the shape of the yield curve under a variety of scenarios. Some frequently used bond strategies involve the use of bullet and barbell bond portfolios. Each of these strategies works best depending on the specific market context. For instance, in order to construct a barbell bond portfolio, the investor needs to buy securities with maturities clustered at two different maturities (i.e., the wings), one at the short end of the yield curve and the other at the long end of the curve, usually in a 50-50 split. This strategy provides a higher yield from the long end and cash flexibility (to eventually purchase other longer-term bonds in an increasing interest rate environment) from the short end. In effect, a barbell creates a medium term average weighted maturity. In a flattening yield curve environment, traders who buy this strategy are anticipating a drop in the longer term yields. A bullet strategy is simply a maturity matching strategy.

An analysis of the relative performance of these two types of bond strategies is complicated by the fact that changes in the slope and curvature of the yield curve are not independent from shifts in the overall level of interest rates and the latter can result from a variety of reasons. Indeed, Litterman and Scheinkman (1991) propose that overall changes in the shape of the yield curve stem from parallel transformations, changes in the slope (i.e. rotations), and changes in the curvature (i.e. concavity of the curve). Generally, an upward shift in the interests level leads to a flattening of the yield curve and a decrease in the curvature, while a downward shift produces a steepening curve and an increase in the curvature. Jones (1991) and Wilner (1996) propose and demonstrate that the parameter changes are not independent from each other. Therefore, bond portfolio strategies, specifically those involving bullet and barbell strategies need to take into account the co-movement patterns among the three yield curve factors.

Another important aspect concerns the specific method employed to fit the yield
Among others, Mann and Ramanlal (1997) employ a quadratic approximation. This method has an intuitive appeal since the level, slope, and curvature measures are easily collected as the coefficients of the polynomial terms. Another possible approach employs the Nelson-Siegel parametrization Nelson and Siegel (1987), which is a parsimonious representation built on the exponential function. Using these methods and taking into account the co-movement among the yield curve factors, several studies (e.g., Jones (1991), Wilner (1996), Mann and Ramanlal (1997)) have analyzed the relative performance of the two bond strategies.

Several results are worth mentioning. For example, Jones (1991) and Mann and Ramanlal (1997) find that downward shifts in the yield curve along with the implied changes in slope and curvature lead to an outperformance of the bullet relative to the barbell portfolio. The opposite outcome occurs when the yield curve shifts upward: the barbell outperforms the bullet portfolio. These findings are robust to the initial shape of the yield curve and to changes in the coupon rate. However, when the terms to maturity change, the relative performance of the bullet and barbell portfolios also changes. At the short end of the yield curve (i.e., for short maturity rates) the finding above regarding the relative performance of the bullet and barbell strategies remains unaltered (i.e., the bullet underperforms the barbell for an upward shift in the level and vice-versa). However, for longer maturities and upwards shifts in the interest rate level, a bullet strategy outperforms a barbell one.

While the papers referenced above have provided important contributions to the literature, several important issues have not been considered yet. For instance, none of the previous studies have looked at the relative performance of bullet and barbell portfolios in tandem with stock market changes (say as measured by the performance of the Dow Jones Industrial Average). There is an ample literature revolving around the correlation between stock and the bond markets. For instance, Pastor and Stambaugh
(2003), Connolly et al. (2005), Christiansen and Ranaldo (2007), and Bansal et al.
(2010) among others investigate the various dynamics of the stock bond correlation and
document the large swings of their co-movement. Thus, the evidence in those studies
shows that the correlation was largely positive throughout the 1980’s and most of the
1990’s and predominantly negative after 2000. In particular, Balduzzi et al. (2001) argue
that macroeconomic announcements have a significant impact on the prices of a three
month bill, a two year note, a 10-year and a 30-year bond, respectively and that public
news can explain a substantial fraction of price volatility in the after math of those
announcements. Since the Dow Jones also reacts quickly to macroeconomic news and
moreover, since a stock market shock may carry implications for bond strategies, one
may expect that changes in the Dow Jones index affect the overall level of interest rates.

Further, none of the studies above have considered the performance of bullet and
barbell strategies linked to the LIBOR swap curve. For example, municipal bond issuers
may choose to utilize ”synthetic” fixed rate structures based on LIBOR rates in order to
create net liabilities at lower interest rates than can be achieved by issuing traditional
fixed rate bonds. Moreover, the studies mentioned above have either stopped short
of reporting the confidence intervals of their point estimates or have used traditional
approaches (i.e., +/- 1.96 times the standard deviation) that may not be entirely appro-
priate given the fat-tails and extreme outliers in yield return data. However, in order to
be able to properly compare our results with those elsewhere in the literature we also
report robustness results using Treasury data.

The current study aims to fill some of the missing gaps in the literature. In addi-
tion to the contributions meant to address the issues listed above, we also employ an
additional parametrization of the yield curve. Thus, in addition to the square poly-
nomial approximation of Mann and Ramanlal (1997), the Nelson-Siegel representation
(i.e., Nelson and Siegel (1987)) and the Svensson (1994) extension of the Nelson-Siegel
approach, we also propose a cubic parametrization. We call the extra term that measures the rate of change of the curvature the surge. Results in the paper show that the surge factor co-moves with the other yield term factors and with the broader market changes. We document that the new yield curve fitting model yields lower root mean squared and lower root mean forecast errors, respectively than the other two approaches. In addition, our proposed model yields tighter bootstrapped confidence intervals than the square polynomial approximation.

Results in this study align with previous ones and show that an upward (downward) shift is accompanied by a flattening (steepening) of the curve and a decrease (increase) in the curvature. Those results are robust to various choices for the initial yield curve and across various coupon rates. New findings show that extreme stock market shocks in excess of 15% in either direction have a higher impact than stock market shocks in between those values. We also find that the choice of the yield curve parametrization carries significant weight for the qualitative interpretation of the bond strategies. For instance, using Libor data a Nelson-Siegel parametrization suggests that an extreme negative shock to the stock market, causes the barbell to outperform the bullet portfolio, while the polynomial representation suggests the opposite. Nevertheless, the bootstrapped confidence intervals of each of the three yield curve models display roughly the same general pattern. Thus, the evidence in this paper suggests that in the presence of an extreme negative market shock, the attractiveness of the barbell portfolio increases relative to that of the bullet strategy. The opposite outcome results when there is an extreme positive market shock. We assess the soundness of our findings by performing several robustness tests. Hence, the findings of the current study point to the important role that the choice of the yield curve parametrization plays when assessing the relative performance of bullet and barbell strategies.

The rest of the paper is structured as follows. Second 2 summarizes the three different
parameterizations and details the estimation approach of the changes in the yield curve factors where we allow for parameter co-movement. Section 3 demonstrates the superior in-sample and out-of-sample fit of the new method. Section 4 performs the bootstrap algorithm, generates the confidence bands of the relative performance of the bullet and barbell portfolios, assesses their shape relative to various stock market changes, terms to maturity, and coupon rates and conducts a couple of robustness tests. Finally, section 5 concludes.

1 Yield Curve Fitting Models

In the sections below we decide to obtain our main results using the Libor dataset due to the larger and more complete sample. Thus, for the Treasury data there is a gap in the series of the 20-year maturity between January 1987 and October 1993 and a gap in the series of the 30-year maturity between March 2002 and March 2006. However, we use a sample of Treasury data over the October 1993 and December 2012 period to assess the robustness of our findings. The results across the two datasets appear to be relatively similar.

1.1 Nelson-Siegel Parameterization

Following Nelson and Siegel (1987), Diebold and Rudebusch (2012) propose a dynamic\(^1\) version of the instantaneous forward rate curve:

\[
f_t(\tau) = \alpha_t + \beta_t e^{-\tau/h} + \gamma_t \frac{\tau}{h} e^{-\tau/h} + \epsilon_t
\]

\(^1\)As Diebold and Rudebusch (2012) argue, the dynamic Nelson-Siegel curve they propose is actually a different factorization from the one originally proposed by Nelson and Siegel (1987) who use \(y_t(\tau) = b_{1t} + b_{2t} \left( \frac{1-e^{-\lambda \tau}}{\lambda \tau} \right) - b_{3t} e^{-\lambda \tau} \). See Diebold and Rudebusch (2012) (pp. 23-30) for more details.
which implies the following dynamic representation for the yield curve:

\[ y_t(\tau) = \alpha_t + \beta_t \frac{1 - e^{-\tau/h}}{\tau/h} + \gamma_t \left( \frac{1 - e^{-\tau/h}}{\tau/h} - e^{-\tau/h} \right) \] (2)

Above, \( \tau \) denotes the maturity of a regular coupon bond, \( h \) refers to the yield curve hump positioning, and \( \alpha_t, \beta_t, \) and \( \gamma_t \), denote the level, slope, and curvature parameters. Diebold and Li (2006) show that since the loading on \( \alpha_t \) is a constant, then \( \alpha_t \) can be viewed as a long term factor. Further, since the loading on \( \beta_t \), the function \( \frac{1 - e^{-\tau/h}}{\tau/h} \) starts at 1 and decays monotonically to zero, suggests that \( \beta_t \) can be viewed as a short term factor. Finally, the loading on \( \gamma_t \), \( \frac{1 - e^{-\tau/h}}{\tau/h} - e^{-\tau/h} \) starts at zero reaches a maximum and then decays to zero and thus can be viewed as medium term factor. Diebold and Li (2006) show that the coefficients corresponding to three factors outlined above can be interpreted as the level, slope, and curvature of the yield curve. We find the hump shape parameter (i.e., \( \lambda = 1/h \)) by searching for the value in the range from 0.001 to 20 in increments of 0.001 that minimizes the average root mean squared error (RMSE) over the full sample. Exhibit (1) shows that the optimal value is 3.861 (\( h = 1/\lambda \)) close to that employed by Wilner (1996).\(^2\)

Svensson (1994) proposes an extension of the Nelson-Siegel approach to include an extra hump-shape term and to increase flexibility and improve the fit:

\[ y_t(\tau) = \alpha_t + \beta_t \frac{1 - e^{-\tau/h_1}}{\tau/h_1} + \gamma_t \left( \frac{1 - e^{-\tau/h_1}}{\tau/h_1} - e^{-\tau/h_1} \right) + \omega_t \left( \frac{1 - e^{-\tau/h_2}}{\tau/h_2} - e^{-\tau/h_2} \right) \] (3)

Please note that the factorization above is a "dynamic" version of the approach that Svensson (1994) proposes. As we show below, while the modified version of Svensson (1994) does appear to provide a better out-of-sample fit in terms of a lower root mean

\(^2\)Wilner (1996) uses a value of 3 for \( h \).
square forecast error (i.e., RMSE or RMFE are used interchangeably throughout the paper), we focus on the dynamic Nelson-Siegel model due to its parsimony and popularity in the literature and in particular among central banks\(^3\).

\[\text{Exhibit 1: Optimal Nelson-Siegel hump-shape parameter}\]

We find the optimal \(\lambda\) by simulating the yield curve fit using the Nelson-Siegel parametrization over the range \([0.001, 20]\) using increments of 0.001 and choosing the optimal hump parameter as the value that minimizes the root mean square error.

As one can notice, the choice of \(h\) within the \([0,16]\) interval does not seem to matter significantly for the RMSE behavior. The surface of the RMSE is relatively flat in that respective region. Therefore, the yield curve fit using the Nelson-Siegel approach should be robust to our choice of \(h\).

Similarly, we follow the same procedure to find the two parameters for the Svensson modification of the dynamic Nelson-Siegel factorization. Thus, using a search algorithm from 0.1 to 5 in steps of 0.1 we find the optimal values to be \(h_1 = 5\) and \(h_2 = 2.5\).

\(^3\)For example, the BIS (2005) report points out the central banks in a number of countries including France, Germany, and Spain all use the NS model to model the yield curve.
1.2 A Square Polynomial Parametrization

The square polynomial approximation has been used before among others by Mann and Ramanlal (1997). Using the notation above, the yield curve equation writes in the following way:

\[ y_t(\tau) = \alpha_t + \beta_t \tau + \gamma_t \tau^2 + \epsilon_t \]  

(4)

Given our data, \( \tau \) varies from 1 to 12 months and \( t \) from 1 to 6477 daily observations. It is immediately clear why the \( \alpha_t \), \( \beta_t \) and \( \gamma_t \) approximate the level, slope and curvature. Our analysis below shows that the yield curve level using this parametrization is relatively close to the one from the Nelson-Siegel approximation. However, the two approximations yield values for the slope and curvature terms that are quite different from each other. We will return to this issue in the second section.

1.3 A Cubic Polynomial Parametrization

Yield curve fitting simulation exercises below show that the square polynomial representation, while convenient and straightforward to understand, provides a higher RMSE than the Nelson-Siegel one. Therefore, we seek to improve upon the second parametrization and propose the following yield curve fitting model:

\[ y_t(\tau) = \alpha_t + \beta_t \tau + \gamma_t \tau^2 + \theta_t \tau^3 + \epsilon_t \]  

(5)

As one can notice, we introduce a cubic term that we call the \textit{surge}. This coefficient, which corresponds to the third derivative of the yield curve function with respect to the maturity level, can be interpreted as the rate of change of the curvature. It is routinely used in physics for instance to denote the rate at which acceleration is changing (i.e.,
the jerk) and from a geometric standpoint, the surge denotes the osculating parabola in the vicinity of a point. In our case, we interpret the surge as the rate of change of the hump shape of the yield curve, as the latter flattens or steepens.

An investigation of the time series properties of the surge reveals that this additional yield curve factor is a stationary process with a high frequency component. The distribution of this factor is characterized by a mean close to zero, with volatility that is almost four times higher than its mean, and large and positive skeweness and kurtosis coefficients, respectively. An inspection of the correlogram reveals that this yield curve factor is a relatively persistent process, with the degree of persistence increasing significantly since 2008. Finally, although the volatility of this term appears to have decreased significantly over time it seems that it spikes prior to business cycle changes, including the such most recent change at the end of 2007.

Other factorizations could be considered, including spline-based\textsuperscript{4}. However, due to brevity and space considerations we limit to the choices considered above.

1.4 Modeling the Co-movement of the Yield Curve Factors

Following Jones (1991), Wilner (1996), and Mann and Ramanlal (1997), we allow the slope, curvature, and surge terms to respond to changes in the yield curve level. For instance, to evaluate the changes in the yield curve factors over a time window of \( s \) days, we compute \( \Delta \alpha_{t,s} = \alpha_t - \alpha_{t-s}, \Delta \beta_{t,s} = \beta_t - \beta_{t-s}, \Delta \gamma_{t,s} = \gamma_t - \gamma_{t-s}, \text{ and } \Delta \theta_{t,s} = \theta_t - \theta_{t-s}. \)

Since we rule out the independence of the yield curve factors, we propose that the slope, curvature, and surge terms respond linearly and contemporaneously to changes in the yield curve level: \( \Delta \beta_{t,s} = a_1 + b_1 \Delta \alpha_{t,s} + u_{t,s}, \Delta \gamma_{t,s} = a_2 + b_2 \Delta \alpha_{t,s} + u_{t,s}, \text{ and } \Delta \theta_{t,s} = a_3 + b_3 \Delta \alpha_{t,s} + u_{t,s}. \)

\textsuperscript{4}The cubic spline for instance has the well known disadvantage that estimates of forward rates may be rather unstable especially for the longer maturities. See Shea (1984) for more details.
To demonstrate that shifts in the level affect the other yield curve factors, we compute the 51-day rolling window cross-correlations among the empirical level, slope, curvature, and surge terms, respectively.

Following the literature, we set the longest maturity, which in our case is the 12-month Libor rate as the level (i.e., $\alpha_t = y_t(12)$). Next, we set the difference between the 6-Month and the 1-Month Libor rates (i.e., $\beta_t = y_t(6) - y_t(1)$) as the slope and the difference between twice the 4-Month rate and the sum of the 2-Month and 6-Month rates as the curvature (i.e., $\gamma_t = 2y_t(4) - (y_t(2) + y_t(6))$), respectively. Finally, we approximate the surge term as the difference between the sum of the 12-Month and the 1-Month less twice the 6-Month rate and the sum of the 11-Month and 2-Month less twice the 5-Month rate, respectively (i.e., $\theta_t = (y_t(12) + y_t(1) - 2y_t(6)) - (y_t(11) + y_t(2) - 2y_t(5))$). Exhibit (2) shows that the cross-correlation coefficients are significantly large and exhibit bouts of extreme co-movement over time.

Further, Exhibit (3) shows that the average 51-day\(^5\) moving correlation coefficients among the various yield curve factors are relatively large and significantly different from zero, even though the correlation coefficients over the full sample are markedly smaller.

Indeed, a comparison of the empirical values of the yield curve factors with the artificial ones (based on the three yield curve fitting models), confirms the validity of our proposed measures. For example, the correlation between the empirical and the artificial levels varies between 0.95 for the Nelson-Siegel factorization and 0.98 for the square polynomial one.

The correlation between the empirical and the artificial slope is 0.97, 0.86, and -0.83 for the square and cubic polynomial and the Nelson-Siegel representation, respectively.

The co-movement between the empirical and the artificial curvature terms exhibits sim-

\(^5\)We have experimented with other moving blocks, like 381 days for instance, without significant changes of the main qualitative findings. These additional results are available per request from the authors.
Rolling Correlation Coefficients Among the Yield Curve Factors

Using equal windows of data between 1/2/1987 and 1/24/2013

Exhibit 2: Rolling Cross-Correlations

This exhibit shows the 51-day rolling window cross correlations of the four yield curve factors. These are empirical yield curve factors, which are obtained in the following way: we set the level at the longest maturity possible (i.e., \( y_t(12) \)); we fix the slope as difference between the 6-Month and the 1-Month Libor rates (i.e., \( y_t(6) - y_t(1) \)); we fix the curvature as the difference between twice the 4-Month rate and the sum of the 2-Month and 6-Month rates (i.e., \( 2y_t(4) - (y_t(2) + y(6)) \)), respectively. Finally, we approximate the surge term as the difference between the sum of the 12-Month and the 1-Month less twice the 6-Month rate and the sum of the 11-Month and 2-Month less twice the 5-Month rate, respectively (i.e., \( (y_t(12) + y(1) - 2y(6)) - (y_t(11) + y_t(2) - 2y(5)) \)).
Exhibit 3: Cross Correlations among Empirical Yield Curve Factors

<table>
<thead>
<tr>
<th>Correlation Coefficients of</th>
<th>Full Sample window (mean)</th>
<th>51-day moving window (std. error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level and Slope</td>
<td>-0.11</td>
<td>0.69</td>
</tr>
<tr>
<td>Level and Curvature</td>
<td>-0.02</td>
<td>0.50</td>
</tr>
<tr>
<td>Level and Surge</td>
<td>0.06</td>
<td>0.69</td>
</tr>
<tr>
<td>Slope and Curvature</td>
<td>-0.73</td>
<td>0.69</td>
</tr>
<tr>
<td>Slope and Surge</td>
<td>0.18</td>
<td>0.46</td>
</tr>
<tr>
<td>Curvature and Surge</td>
<td>-0.29</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Exhibit 4: Correlation between Empirical and Artificial Yield Curve Factors

<table>
<thead>
<tr>
<th>Correlation Coefficients of</th>
<th>Square Polynomial</th>
<th>Cube Polynomial</th>
<th>Nelson Siegel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empirical and Artificial Level</td>
<td>0.9822</td>
<td>0.9728</td>
<td>0.9513</td>
</tr>
<tr>
<td>Empirical and Artificial Slope</td>
<td>0.9728</td>
<td>0.8597</td>
<td>-0.8501</td>
</tr>
<tr>
<td>Empirical and Artificial Curvature</td>
<td>0.9881</td>
<td>0.8389</td>
<td>-0.6705</td>
</tr>
<tr>
<td>Empirical and Artificial Surge</td>
<td>0.4441</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

the next section argues that the extended polynomial model fits the data better using both in-sample and out-of-sample measures, respectively. This finding will matter when assessing the relative performance of yield curve strategies.

2 Optimal Yield Curve Approximation

The ability of various yield curve fitting methods to interpolate among the missing maturities plays an important role when comparing barbell and bullet strategies. Therefore, this section investigates the in-sample and out-of-sample forecasting performance of the three measures we propose.

First, we compute the daily in-sample RMSE and then average the results using each method. We find an average RMSE of 10.22% when we use the square polynomial approach, a RMSE of 7.60% when we employ the cubic polynomial method, and a RMSE of 9.01% when we fit with the Nelson-Siegel approach, respectively. These findings
suggests that including the extra parameter pays off by improving the fit. As previous research has already shown (e.g., Wilner (1996)), each method on average explains 95% of the variance of the dependent variable.

Next, we compare the artificial level, slope, and curvature factors using the three approaches. Exhibits (5) and (6) have the results.

**Exhibit 5: Descriptive Statistics of Artificial Yield Curve Factors**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Error</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level (Sq)</td>
<td>4.18</td>
<td>2.63</td>
<td>0.048</td>
<td>10.143</td>
</tr>
<tr>
<td>Level (Cubic)</td>
<td>4.16</td>
<td>2.63</td>
<td>-0.105</td>
<td>10.132</td>
</tr>
<tr>
<td>Level (Nelson-Siegel)</td>
<td>4.98</td>
<td>2.45</td>
<td>0.96</td>
<td>12.009</td>
</tr>
<tr>
<td>Slope (Sq)</td>
<td>0.046</td>
<td>0.076</td>
<td>-0.308</td>
<td>0.372</td>
</tr>
<tr>
<td>Slope (Cubic)</td>
<td>0.062</td>
<td>0.123</td>
<td>-0.602</td>
<td>0.769</td>
</tr>
<tr>
<td>Slope (Nelson-Siegel)</td>
<td>-0.783</td>
<td>0.791</td>
<td>-3.204</td>
<td>1.691</td>
</tr>
<tr>
<td>Curvature (Sq)</td>
<td>-0.0006</td>
<td>0.003</td>
<td>-0.018</td>
<td>0.016</td>
</tr>
<tr>
<td>Curvature (Cubic)</td>
<td>-0.004</td>
<td>0.015</td>
<td>-0.085</td>
<td>0.070</td>
</tr>
<tr>
<td>Curvature (Nelson-Siegel)</td>
<td>-0.696</td>
<td>1.049</td>
<td>-3.455</td>
<td>3.488</td>
</tr>
<tr>
<td>Surge</td>
<td>0.0002</td>
<td>0.0006</td>
<td>-0.003</td>
<td>0.004</td>
</tr>
</tbody>
</table>

The Exhibits above show that overall the level approximations follow each other closely, irrespective of the measure used. In particular, the levels generated by the square and cubic polynomial are virtually identical. However, the Nelson-Siegel approximation appears to generate a level measure that is biased upwards relative to the previous two methods.\(^6\) Also, one can note that the cubic polynomial generates for a brief period negative values for the level in the aftermath of the Great Recession. More sharp differences result in the case of the slope and curvature measures, respectively. Thus, on average the Nelson-Siegel approach yields a negative value for the slope. The volatility of the slope measure also appears at least six times higher than that of the other two measures. The Nelson-Siegel curvature measure also appears much higher in absolute value with a significantly higher volatility than those of the other two methods.

\(^6\)We also estimate the level, slope and the two curvature parameters for the Svensson approach. They are 5.55, -1.41, -1.69, and 0.65, respectively. These values are similar to the ones of the dynamic Nelson-Siegel factorization.
The panels show each of the yield curve factors obtained using three possible yield curve parameterizations.

To further investigate the yield curve fit and the forecasting performance of the three methods, we also perform an out-of-sample forecasting exercise. Precisely, we forecast the one step ahead yield curve rates from the one month to the twelve month rate, respectively using a 51-day moving average of the previous fitted rates. We also employ a random walk model (i.e., naive forecasting) as the benchmark. At each step we compute the one step ahead root mean square error and then take the average over the full sample. Unfortunately, we find that none of the yield curve parameterizations is able to outperform the naive forecasting model. Nevertheless, the out-of-sample forecasting exercise reinforces the usefulness of our proposed cubic polynomial since its root mean forecast error (RMFE) is the lowest (at 21.13%) among the the three models.\footnote{The random walk model yields an average RMFE of 21.12\%, the quadratic approximation yields...}
3 Bootstrapping Bullet and Barbell Strategies

3.1 The Impact of Market Changes on the Yield Curve Level

The evidence above suggests that shifts in the yield curve level are accompanied by changes in the slope, curvature, and surge terms. Therefore, an analysis of the relative performance of bullet and barbell strategies requires the modeling of the co-movements among the yield curve factors. In addition to results elsewhere in the literature, we consider how shocks to the stock market performance as indicated by the closely watched Dow Jones index affect bond portfolio strategies. We classify the Dow Jones shocks into several categories, depending on the percent change in the index’s return: extremely negative when the Dow falls by more than 15% over a six month period, strongly negative when the Dow falls between 15% and 10%, moderately negative when the Dow falls between 10% and 5% and mildly negative when the Dow falls between 0% and 5%. Further, we classify a Dow Jones move as mildly positive when the market change is between 0 and 5%, moderately positive when the market moves between 5% and 10%, strongly positive when the market move is between 10% and 15%, and finally extremely positive when the market advances by more than 15%.

The sample period includes daily LIBOR and Dow Jones data from January 2\textsuperscript{nd} 1987 until January 24\textsuperscript{th}, 2013. Overall, there are 312 days (4.91% of the sample) in which the Dow Jones decreased by more than 15%, 257 days (4.05% of the sample) in which it decreased between 15% and 10%, 469 days (7.38%) in which it decreased between 10% and 5%, and 876 days (or 13.79% of the sample) in which it moved downwards by up to 5%. Further, there were 1375 instances (or 21.65% of the sample) where the market advanced by less than 5%, 1209 days (19.03% of the time) where the market advanced an average RMFE of 21.19% and the Nelson-Siegel parametrization yields an average out-of-sample RMFE of 21.17%, respectively. The Svensson (1994) factorization yields an out-of-sample RMFE of slightly more than 21.13%.
between 5% and 10%, 909 days (14.31% of the time) where the Dow Jones increased between 10% and 15%, and 945 days (14.88% of the time) when the Dow Jones expanded by more than 15%. As expected, the correlation between the Dow Jones performance and the yield curve level (as approximated by the square polynomial approach) for the full sample is negative and relatively large at 58.89%. This aligns with the historical evidence that on average over the last couple of decades, stock market movements and overall interest rates levels have been negatively correlated.

In equations (5) through (11) we follow the previous literature and set $s = 6$ months\(^8\) of daily observations. We regress $\Delta \alpha$ on a set of eight binary variables to capture the various Dow Jones movements we consider:

$$\Delta \alpha = c_1 D(r_t < -15%) + c_2 D(-15% < r_t < -10%) + c_3 D(-10% < r_t < -5%) + c_4 D(-5% < r_t < 0%) + c_5 D(0 < r_t < 5%) + c_6 D(5% < r_t < 10%) + c_7 D(10% < r_t < 15%) + c_8 D(15% < r_t) + \epsilon_t$$ (6)

For instance, in the case of the quadratic approximation we obtain the following estimates over the full sample:

$$\Delta \alpha = -0.5691 D(r_t < -15%) - 0.9392 D(-15% < r_t < -10%) - 0.7505 D(-10% < r_t < -5%) - 0.1215 D(-5% < r_t < 0%) + 0.2621 D(0 < r_t < 5%) - 0.0004 D(5% < r_t < 10%) - 0.0994 D(10% < r_t < 15%) - 0.2204 D(15% < r_t) + \epsilon_t$$ (7)

\(^8\)We have also experimented with $s = 1$ to 12 months of daily observations. However, the results we obtain do not appear to be affected by the particular choice of $s$. 

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With the exception of $c_6$, all other coefficients are highly significant at the one percent significance level.\(^9\) Several remarks are warranted. First, with the exception of the $[0,5\%]$ range, all coefficients are negative. This result implies that most of the time stock market shocks have a negative impact on the interest rate level irrespective of their sign. Second, we have that in seven cases a market condition implies that $\Delta \alpha_t < 0$, while only over the range $[0,5\%]$ we have that $\Delta \alpha_t > 0$. Third, we may note that a negative stock market shock has a higher impact in absolute value than a positive one.

However, we should point out that bootstrapping the equation above 100,000 times and averaging the results, we find that only the more severe market movements display significant coefficients at the 66% significance level.

**Exhibit 7:** Impact of Market Condition on the Change in the Interest Rate Level

![Exhibit 7](image)

The panels above show the bootstrapped confidence intervals of the coefficients measuring the impact of the market conditions on the change in $\Delta \alpha_t$. The left panel shows the coefficients from the cubic polynomial approximation, while the right panel shows the coefficients from the Nelson-Siegel approximation.

Thus, Exhibit (7) shows that irrespective of the yield curve method used, an extreme negative market movement has a significantly positive impact on the change in level at the 66% level, while an extreme positive market change has the opposite effect. The latter result contrasts with the impact above over all the sample.

To reconstruct the yield curve, several ingredients are needed: the expected changes

\(^9\)A re-estimation of this equation using the cubic polynomial and the Nelson-Siegel approximation, respectively yields similar coefficients. Thus, they all exhibit negative values with only one exception for the $[0,5\%]$ range.
of the slope and the curvature given a specific change in the level, where the latter depends on the particular shock to the Dow Jones index. Then, the expected shape of the yield curve, given an initial state of the world \((\alpha_0, \beta_0, \gamma_0)\) and denoted by \(y(\tau)\) is given by:

\[
y(\tau) = (\alpha_0 + E(\Delta \alpha_0|r_t)) + [\beta_0 + E(\Delta \beta_0|E(\Delta \alpha_0|r_t))]|\tau + [\gamma_0 + E(\Delta \gamma_0|E(\Delta \alpha_0|r_t))]|\tau^2 \tag{8}
\]

We proceed in a similar manner for the other two approaches. In particular, for the cubic polynomial we have an additional expectation that involves the surge term.\(^{10}\)

To assess the usefulness and the practical implications of our approach, we use the equation above to recompute the yield curve using the Dow Jones market changes. We perform this exercise out-of-sample and compare the yield curve forecasts with those obtained without the Dow Jones index changes and simply using the polynomial, dynamic Nelson-Siegel and Svensson parameterizations, respectively.

As Table (8) shows, incorporating the changes in a stock market index like the Dow Jones can in fact improve the out-of-sample forecast of interest rates when using the out-of-sample root mean square forecast error as a criterion. For instance, the evidence shows that on average over the early period of 1987-1994 the information in the stock market index can yelp improve the forecast of the yield curve relative to not using such information. The same conclusion emerges when investigating the period after the recent financial crisis. However, it appears that over the 1995-2001 and 2002-2007 periods, the information in the stock market does not appear to improve the forecasts of interest rates

\(^{10}\)The expectation of the surge term writes as \([\theta_0 + E(\Delta \theta_0|E(\Delta \alpha_0|r_t))]|\tau^3\). To save space, we omit reporting the coefficient estimates for those other two approaches.
Exhibit 8: Yield curve forecasts with and without the Dow Jones changes

<table>
<thead>
<tr>
<th>DJIA changes</th>
<th>Quadratic</th>
<th>Cubic</th>
<th>Nelson-Siegel</th>
<th>Svensson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>RMFE</td>
<td>RMFE</td>
<td>RMFE</td>
<td>RMFE</td>
</tr>
<tr>
<td>1987 - 1994</td>
<td>1</td>
<td>0.981</td>
<td>1</td>
<td>0.984</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1995 - 2001</td>
<td>1</td>
<td>1.037</td>
<td>1</td>
<td>1.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002 - 2007</td>
<td>1</td>
<td>1.165</td>
<td>1</td>
<td>1.092</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008 - 2013</td>
<td>1</td>
<td>0.991</td>
<td>1</td>
<td>0.971</td>
</tr>
</tbody>
</table>

This table shows the out-of-sample root mean forecast error of the yield curve with and without Dow Jones index changes taken into account. We set the RMFE of each model when the DJIA changes are not considered as the benchmark. We use the previous 126-day period stock market changes and yield curve information to estimate each of the yield curve factorization parameters and the average DJIA changes.

beyond the simple parameterizations involving polynomials or the Nelson-Siegel based ones. Overall, the evidence in Table (8) points to the fact that stock market changes do affect the yield curve parameters via shocks to the yield curve level. These results are consistent across each of the four possible yield curve parameterizations.

3.2 Evaluating Portfolio Performance

We employ a three step process to assess the relative performance of the bullet and barbell strategies. First, we need to compute the value of the portfolio given an initial yield curve shape ($\alpha_0$, $\beta_0$, $\gamma_0$, and $\theta_0$, respectively) while assuming that $E(\Delta \alpha_0 | r_t) = 0$. The first step yields the portfolio’s benchmark return. The second step computes the shape of the yield curve given the various anticipated market conditions over the interval s. Lastly, the third step assesses each portfolios value after the anticipated market condition has impacted the yield curve level. We experiment with different initial yield curves to assess the importance of the initial conditions. Thus, we consider as the starting point a relatively low, medium, and high yield curve level, respectively. We also
investigate the impact of different coupon rates and of different terms to maturity of the bullet portfolio.

The bullet and barbell portfolios are evaluated by finding the present value of the future cash flows that they generate in the presence of several yield curve transformations. The construction of the portfolios is analogous to that of Mann and Ramanlal (1997). For instance, we center a bullet portfolio at a specific maturity that makes a fixed coupon payment. We allow the coupon rates to vary between 2% and 10%. Finally, we consider terms to maturity of four to nine months. The dollar amount invested in the bullet is multiplied by two in order to correspond to the two bonds in the barbell portfolio. The nominal amount need not be the same, as we are purely interested in the change in the overall values of each portfolio. However, the resulting bullet and barbell portfolios display on average identical weighted durations.

The initial question we examine is which portfolio (bullet or barbell) provides superior returns given an anticipated market condition that leads then to changes in level, slope and curvature. We explore the relative performance of the portfolios using the eight different market expectations mentioned above. The study is conducted using (i) 52 of the 6477 days as the initial yield curves, sampling every last day in May and December, (ii) seventeen different coupon rates from 2% to 10% in 0.5% increments, (iii) six different terms to maturity in one month increments.

We also compute the confidence intervals by employing a moving-block bootstrap where the size of the block is 381 days. The bootstrap approach uses a fixed block length with overlapping blocks following the suggestions in the literature (see Davidson

\[11\] For instance, when the bullet portfolio has a 6-Month maturity, the wings have maturities at the 3rd and 9th month, respectively. The bullet par value is $2,000, while the par values of the wings are each $1,000.

\[12\] In results not reported in this paper but available per request from the authors, we find that as we increase the size of the moving block we get fewer observations in the tails and the confidence intervals in the tails get wider. Therefore, as the sampling window size is increasing the degree of uncertainty increases, which renders that the bullet and barbell portfolios yield a similar performance.
and Mackinnon (2004)). We have experimented with several data windows such that
(i) we obtain an integer number of blocks and (ii) we properly account for the serial
correlation in returns. Thus, blocks of 51 days and 127 days appear to yield noisy results
and to be insufficiently long to capture the long-memory of returns. In contrast, longer
blocks such as the 2159 days block appear to induce little variation in the bootstrapped
results and unrealistically long to capture the serial correlation in returns. Therefore,
we settle for the 381-day window which appears sufficient to capture the autocorrelation
in the data and provide sufficient variation in the simulation outcome. We simulate the
data 100,000 times and compute the 66% confidence intervals. Repeated results show
that the choice of the initial yield curve does not matter for our results. Therefore, the
final simulations employ a medium level as the initial condition of the yield curve.

Exhibits (9), (10), and (11) show the bootstrap results.

Exhibits (9) and (10) display the results from the square and cubic polynomial ap-
proaches, respectively. On the vertical axis we plot the difference between the values of
the bullet and barbell portfolios. On one axis we display the range of the terms to ma-
turity, while on the other axis we fix the eight possible market conditions. Both exhibits
appear to suggest that when changes in the yield curve level are generated by extreme
market movements in excess of 15% on either side, then the actual data lie outside the
66% confidence interval and the outcomes are statistically significant. Thus, an extreme
negative market change appears to yield a confidence interval that lies below the full
sample impact, while at the opposite end, the confidence interval is above the full sam-
ple estimate. Over the full sample, the two figures indicate that a bullet outperforms
the barbell when there is a severe market correction in either direction. However, the
bootstrapped confidence intervals appear to suggest that 66% of the time for a drop of
15% or more in the index, the barbell actually outperforms the bullet strategy. Only
for a surge in the index of more than 15% do we get that the bullet outperforms the
Exhibit 9: Impact of Market Conditions under a Square Polynomial Parametrization

This Exhibit shows the relative performance of barbell and bullet portfolios in the presence of several market conditions modeled as percent changes of the Dow Jones index along with the the terms to maturity. The 66% bootstrapped confidence intervals are obtained using moving blocks of 381 days repeated 100,000 times.

Further, the Exhibits shows that in the $[0,5\%]$ market movement range which corresponds to $\Delta \alpha_t > 0$, the barbell appears slightly superior and this result is statistically significant. This finding confirms the result in Mann and Ramanlal (1997). The findings above hold regardless of the initial yield curve shape or of the coupon rate. One may also note that the bullet outperforms the barbell at an increasing rate as the term to
Exhibit 10: Impact of Market Conditions under a Cubic Polynomial Parametrization

This Exhibit shows the relative performance of barbell and bullet portfolios in the presence of several market conditions modeled as percent changes of the Dow Jones index along with the terms to maturity. The 66% bootstrapped confidence intervals are obtained using moving blocks of 381 days repeated 100,000 times.

maturity shortens. On the other hand, the confidence interval also increases for shorter maturities, especially for those less than six months. However, while the two Exhibits display a similar shape, we should point out that around 88% of the time the cubic approximation yields tighter confidence intervals than the quadratic one. The average savings due to the uncertainty risk reduction corresponding to a $100 billion dollar bond portfolio are approximately $550,000.

Finally, Exhibit (11) appears to generate findings that are quite different from those two above. For example, in contrast to the findings from the polynomial approximations, the full sample results using a Nelson-Siegel representation suggest that a barbell outperforms the bullet across all types of negative market changes. The only instance where the bullet portfolio return is superior corresponds to the $[0,5\%]$ range or when $\Delta \alpha_t > 0$.

However, the bootstrapped confidence intervals yield quite different implications.
Exhibit 11: Impact of Market Conditions and Time To Maturity under the Nelson-Siegel Parametrization

This Exhibit shows the relative performance of barbell and bullet portfolios in the presence of several market conditions modeled as percent changes of the Dow Jones index along with the the terms to maturity. The 66% bootstrapped confidence intervals are obtained using moving blocks of 381 days repeated 100,000 times.

Thus, it appears that for all possible market changes, the 66% confidence intervals include zero (i.e., bullet and barbell strategies yield non-significant price differentials). In addition, with the exception of the extreme positive market shock, for all other market changes the full sample outcome lies outside the confidence intervals and as in the case of the more severe market corrections, it does so by an extensive margin.

Therefore, not only do the full sample findings appear to indicate conflicting results between the polynomial approximations and the Nelson-Siegel one, but also the bootstrapped 66% confidence intervals are at odds among the three parameterizations. Thus, while the full sample polynomial approximations indicate that for both extreme positive and negative market shocks the bullet outperforms the barbell, the bootstrapped confidence intervals indicate that an extreme negative shock yields the opposite outcome. In contrast, the full sample outcome using the Nelson-Siegel approximation shows that
with the exception of the [0,5\%] range where the bullet outperforms the barbell, the barbell’s performance is superior everywhere else. However, the 66% confidence intervals of the latter approach do not yield significant differences between the two portfolios. The concluding impression is that a practitioner needs to be very cautious when selecting the proper yield curve fitting model since the polynomial models and the Nelson-Siegel one yield outcomes that are quite different from each other.

3.3 Robustness Tests

To assess the robustness of our findings we perform three sets of additional results. To fix ideas and following the notation in MacKinnon (2005), let $\theta$ denote a test statistic, and let $\hat{\theta}$ denote the realized value of $\theta$ for a particular sample of size $T$. The statistic $\theta$ is assumed to be asymptotically pivotal, as in Beran (1988). We denote by $F(\theta)$ the cumulative distribution function, or CDF of $\theta$. Normally, a bootstrap DGP is used to generate $B$ bootstrap samples, each of which is used to calculate a bootstrap test statistic $\theta^*_j$ for $j = 1, \ldots, B$. We can then estimate $F(\hat{\theta})$ by $F^*_B(\hat{\theta})$, where $F^*_B(\hat{\theta})$ is the empirical distribution function or the bootstrap distribution, of the $\theta^*_j$.

The double bootstrap, as proposed by Beran (1988), can be used to compute either p-values or confidence intervals, that should at least in theory be more accurate than ordinary bootstrap tests. For instance, let $G(x)$ denote the CDF of the distribution that the bootstrap p-values $p^*_j$ follow. If $F(\theta) = F^*(\theta)$ and $B$ approaches infinity, this distribution would simply be standard uniform. However, because in general it will be unknown, one can use the double bootstrap to estimate $G(x)$ by using a second level of bootstrap.

The double bootstrap requires to first generate $B_1$ first-level bootstrap samples that are used to compute bootstrap statistics $\theta^*_j$, $j = 1, \ldots, B_1$. Then, the ordinary bootstrap p-value $p^*(\hat{\theta})$ is computed. The double bootstrap then requires a second-level bootstrap
DGP for each first-level bootstrap sample indexed by \( j \), following the same approach from the first step. Each second-level bootstrap DGP is then used to generate \( B_2 \) bootstrap samples that are used to compute test statistics \( \theta_{jk}^{**} \) for \( k = 1, \ldots, B_2 \). In our case, to find the confidence intervals, we find sequentially the upper and lower limits based on the samples \( B_2 \), and then average over \( B_1 \) to find a new set of upper and lower limits.

To be more precise, we set \( B_1 = 1000 \) samples of the DJIA and then each \( B_1 \) is sampled randomly 100 times (i.e., \( B_2 = 100 \)). We follow the same approach outlined above where we computed the simple bootstrapped confidence intervals, meaning that we sample moving blocks of 381 days in order to capture the serial correlation in returns. To preserve space, we only report the general findings. However, the detailed results are available by request from the authors. Thus, the double bootstrap confidence intervals look roughly similar to the ones we reported above following the S-shape pattern observed in Exhibits (9), (10), and (11), respectively. Therefore, the qualitative findings we reported above regarding the differences in the performance of the bullet and barbell strategies remain unaltered.

In a second set of robustness tests, we generate an artificial set of yield curve rates by keeping the initial coefficients in equation (6) and the DJIA observations fixed and then repeating the bootstrapping exercise on the new sample. The objective of this robustness test is to make sure there are no errors in the bootstrapping mechanism and that we can replicate a known DGP accurately. Indeed, the differences between the original yield curve factors and the simulated ones are negligible. For instance, using the cubic approximation the differences between the true values and the estimated ones are -6.98536E-06, 6.09937E-07, -4.12967E-07, and -4.08524E-08 for the level, slope, curvature, and the surge respectively. Therefore, we can be confident that the fact that

\[ \text{As before, the returns are computed by computing the log difference in the DJIA index at every six months.} \]
full sample estimates are outside the 66% or 95% at the extreme ends of the Dow Jones spectrum of shocks are not spurious or the result of coding mistakes.

In the third set of robustness results, we apply the same methodology on Treasury data to verify whether the general findings above are preserved. Thus, we obtain daily three, six months, one year up until 20 year maturity Treasuries from October 1st, 1993 until December 31st, 2012. Across this period we have 197 days (4.21% of the sample) in which the Dow Jones decreased by more than 15%, 201 days (4.30% of the sample) in which it decreased between 15% and 10%, 415 days (8.88%) in which it decreased between 10% and 5%, and 696 days (or 14.89% of the sample) in which it moved downwards by up to 5%. Further, there were 1047 instances (or 22.41% of the sample) where the market advanced by less than 5%, 696 days (14.89% of the time) where the market advanced between 5% and 10%, 679 days (14.53% of the time) where the Dow Jones increased between 10% and 15%, and 742 days (15.88% of the time) when the Dow Jones expanded by more than 15%. The distribution of the Dow Jones market movements across this period is relatively similar to that for the previous sample.

We follow the same approach we used for the Libor dataset and compute the confidence intervals of the difference in returns between the bullet and barbell portfolios, respectively. We show below the results using the cubic and Nelson-Siegel approximations, respectively.

An inspection of the bootstrapped confidence intervals using the cubic approximation reveals several conclusions. As before, we see that the difference between the bullet and barbell strategies is more pronounced in the presence of extreme market movements. In particular, the confidence interval of the difference between the bullet and barbell strategies appears to suggest that the barbell portfolio outperforms the bullet one at shorter maturities and when the stock market experiences a negative shock in excess of 15% over 6 months. This result follows closely the one using Libor data. In rest however
Exhibit 12: Impact of Market Conditions and Time To Maturity under the Cubic Parametrization

This Exhibit shows the relative performance of barbell and bullet portfolios using Treasury data in the presence of several market conditions modeled as percent changes of the Dow Jones index along with the the terms to maturity. The 66% bootstrapped confidence intervals are obtained using moving blocks of 381 days repeated 100,000 times.

the differences between the two strategies do not appear statistically significant with the exception of the case where the market moves between zero and 5%. Hence, just as in the case of Libor data, for an up level shift in interest rates the barbell portfolio slightly outperforms the bullet one.

Finally, the confidence intervals\(^\text{14}\) in Exhibit (13) show that again the barbell portfolio slightly outperform the barbell strategy in the presence of an extreme negative market movement in excess of 15%. However, in addition to the same finding using the cubic approximation this result holds across all maturities. In addition, the bullet portfolio appears to outperform the barbell strategy when the market moves up by more than 15% over six months. In rest, the differences in returns between the two bond strategies

\(^{14}\text{We also compute the same confidence intervals using the dynamic version of the Svensson (1994) approach. Since the differences relative to the dynamic Nelson-Siegel factorization are minimal we choose not to report them, even though they are available from the authors upon request. For instance, the average difference between the confidence intervals of the two approaches is close to zero at 0.008.}
Exhibit 13: Impact of Market Conditions and Time To Maturity under the Nelson-Siegel Parametrization

This Exhibit shows the relative performance of barbell and bullet portfolios using Treasury data in the presence of several market conditions modeled as percent changes of the Dow Jones index along with the terms to maturity. The 66% bootstrapped confidence intervals are obtained using moving blocks of 381 days repeated 100,000 times.

do not appear statistically different from each other.

4 Conclusion

We use data from the LIBOR swap yield curve and from the Treasury yield curve, respectively to estimate the three characteristics of the curve namely the level, slope and curvature. We follow previous work by Mann and Ramanlal (1997) to explore the correlation between the three yield curve factors while adding the expected change in the interest rate level given an anticipated market condition as a new element. Further, in addition to a square polynomial approximation of the yield curve, we use a cubic parametrization as well as two dynamic extensions of the Nelson-Siegel approach. We call the rate of change of curvature as the surge. Results in the paper show that the cubic parametrization yields the lowest root mean square error among the four possible choices.
and also yields the lowest root mean forecast error based on an out-of-sample forecasting exercise. We also add to the literature by properly accounting for the uncertainty of the point estimates and by computing the 66% confidence intervals using a moving block bootstrap. We set the block size at 381 days to take into account the serial correlation in returns and perform 100,000 draws for each method. We also employ three sets of robustness tests to make sure that the results stand using different approaches and are not the artifact of coding mistakes.

The empirical results can be summarized as follows. First, results were consistent with previous studies that found that an upward (downward) shift is accompanied by a flattening (steeplening) of the curve and a decrease (increase) in the curvature. Second, for extreme negative market movements, the bootstrapped confidence intervals (using all three methods) suggest that the barbell should outperform the bullet portfolio. For extreme positive market movements, the bullet tends to be superior. This result seems to hold true regardless of the initial yield curve or of the coupon rate.

When looking at the impact of the various terms to maturity, we find that for short and moderate maturities the barbell is superior. We also find that the confidence intervals become tighter with an increase in the terms to maturity. In addition, we find that the cubic approximation yields tighter confidence intervals than the square parametrization. Moreover, we note that the Nelson-Siegel full sample result yields different implications relative to the polynomial approaches with respect to both the market conditions and the terms to maturity. For instance, as the market conditions worsen the Nelson-Siegel parametrization applied on the full sample suggests that a bullet strategy underperforms a barbell one. The cubic polynomial suggests the opposite. Further, using either polynomial method, relatively small (up to 5%) positive shocks to the Dow Jones performance lead to the bullet to outperform the barbell. This effect is statistically significant and confirms results elsewhere in the literature.
Overall, we conclude that the choice of the yield curve model carries significant importance in assessing the relative performance of yield curve strategies. This result draws renewed attention to this topic and reinforces the idea that practitioners and applied researchers need to be careful when selecting a yield curve parametrization.
References


